Spring 2022 Math 208 M Midterm 2

NAME (First,Last) : ....

STUDENT ID ....

UW email ....

- Please use the same name that appears in Canvas.
- IMPORTANT: Your exam will be scanned: DO NOT write within 1 cm of the edge. Make sure your writing is clear and dark enough.
- Write your NAME (first, last) on top of every odd page page of this exam.
- If you run out of space, continue your work on the back of the last page and indicate clearly on the problem page that you have done so.
- Unless stated otherwise, you **MUST** show your work and justify your answers.
- Your work needs to be neat and legible.

Problem 1 Let A be a 4×4 matrix with columns c_1, c_2, c_3, c_4. Suppose that by performing a sequence of elementary operations you can reduce A to

$$B = \begin{pmatrix} 1 & 5 & -1 & 0 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

1. Find the rank of A. No justification necessary.

2. Find a basis for row(A), the row space of A.

3. Find a basis for Null(A), the null space of A. Show your work.

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4. Is c_1, c_2, c_4 a basis for col(A), the column space of A ? Justify your answer.

Consider $T : R^4 \to R^4$, $T(\vec{v}) = A\vec{v}$, where A is the matrix from the previous page.

1. Is T onto ? Justify your answer.

2. Is T one to one ? Justify your answer.

Problem 2 This problem has two unrelated parts.

Find the matrices for 2 different linear transformations T_1 and T_2: $R^3 \to R^2$ both having the values $T_1((1,0,0)) = T_2((1,0,0)) = (2,-1)$ and
$T_1((0,1,1)) = T_2((0,1,1)) = (1,0)$ or explain why this is not possible.

Find the matrix of the linear transformation $T : R^2 \to R^2$ that rotates a vector (x,y) 180 degrees. Show your work to explain how you found this matrix..

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Problem 3 This problem has two unrelated parts.

1. Give an example of a 3x3 matrix A such that Null(A)=span $\left(\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right)$, and col(A)=span $\left(\begin{pmatrix}0\\1\\0\end{pmatrix}\right)$, or explain why this is not possible.

2. Give an example of a 3x3 invertible matrix A such that col(A)=span $\left(\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}2\\2\\1\end{pmatrix}\right)$, or explain why this is not possible.