## Spring 2022 Math 208 M Midterm 2

NAME (First,I	Last) :	 	 
STUDENT ID		 	 
UW email		 	 

- Please use the same name that appears in Canvas.
- IMPORTANT: Your exam will be scanned: DO NOT write within 1 cm of the edge. Make sure your writing is clear and dark enough.
- Write your NAME (first, last) on top of every odd page page of this exam.
- If you run out of space, continue your work on the back of the last page and indicate clearly on the problem page that you have done so.
- Unless stated otherwise, you MUST show your work and justify your answers.
- Your work needs to be neat and legible.

**Problem 1** Let A be a  $4 \times 4$  matrix with columns  $c_1, c_2, c_3, c_4$ . Suppose that by performing a sequence of elementary operations you can reduce A to

$$B = \begin{pmatrix} 1 & 5 & -1 & 0 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- 1. Find the rank of A. No justification necessary.
- 2. Find a basis for row(A), the row space of A.
- 3. Find a basis for Null(A), the null space of A. Show your work.

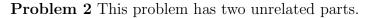
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4. Is  $c_1, c_2, c_4$  a basis for  $\operatorname{col}(A)$ , the column space of A? Justify your answer.

Consider  $T: R^4 \to R^4, \, T(\vec{v}) = A\vec{v}$ , where A is the matrix from the previous page.

1. Is T onto? Justify your answer.

2. Is T one to one? Justify your answer.



Find the matrices for 2 different linear transformations  $T_1$  and  $T_2$ :  $R^3 \to R^2$  both having the values  $T_1((1,0,0)) = T_2((1,0,0)) = (2,-1)$  and  $T_1((0,1,1)) = T_2((0,1,1)) = (1,0)$  or explain why this is not possible.

Find the matrix of the linear transformation  $T: \mathbb{R}^2 \to \mathbb{R}^2$  that rotates a vector (x,y) 180 degrees. Show your work to explain how you found this matrix..

NAME (First,Last):

**Problem 3** This problem has two unrelated parts.

1. Give an example of a 3x3 matrix A such that Null(A)=span  $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ ), and  $col(A)=span \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ ), or explain why this is not possible.

2. Give an example of a 3x3 invertible matrix A such that  $col(A)=span \begin{pmatrix} 1\\0\\1 \end{pmatrix}, \begin{pmatrix} 2\\2\\1 \end{pmatrix})$ , or explain why this is not possible.